■ Interior cells

▨ Boundary cells

Fig. 1 (PRIOR ART)
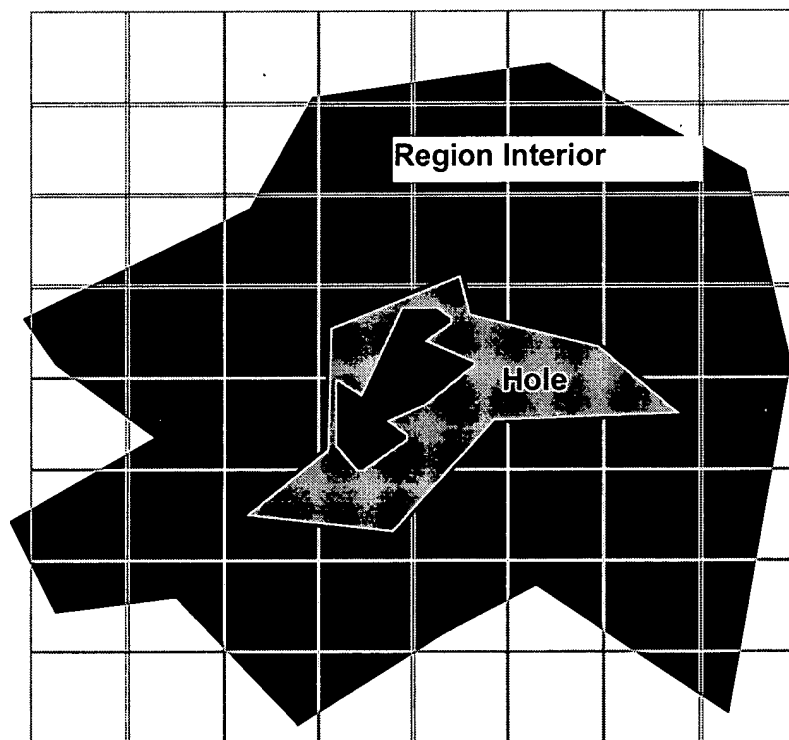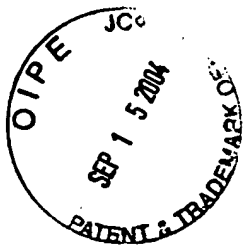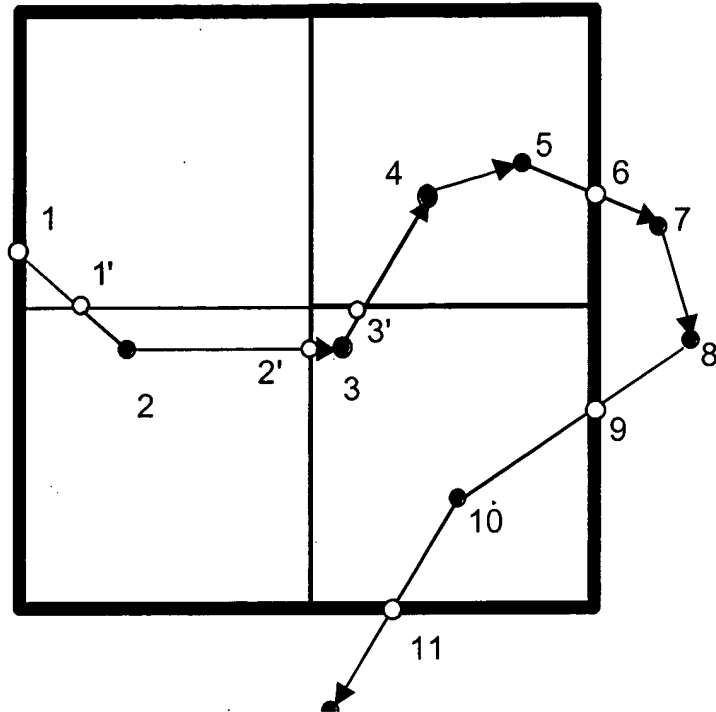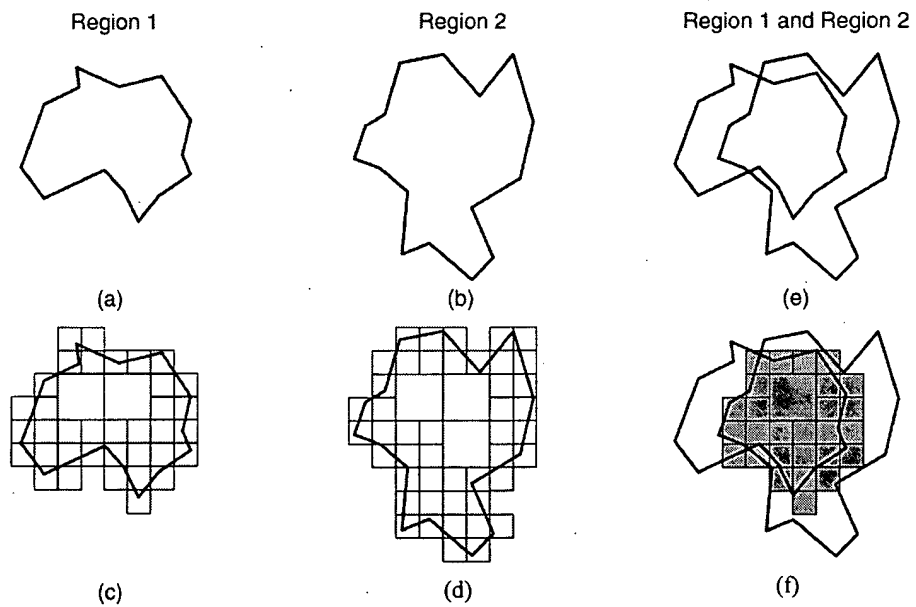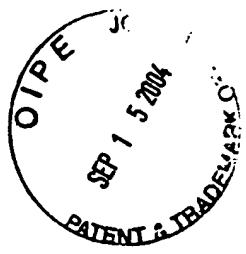
Region Interior

Hole

Fig. 2 (PRIOR ART)

Fig. 3. (PRIOR ART)

Region 1

(a)

(c)

Region 2

(b)

(d)

Region 1 and Region 2

(e)

(f)

Fig. 4 (PRIOR ART)

Legend:
= interior x boundary
= boundary x boundary
= interior x interior

Fig. 5 (PRIOR ART)

Fig. 6

Fig. 7 (PRIOR ART)

$(0, y_{max})$

$(-x_{max}, 0)$

$(x_{max}, 0)$

Interior

$(0, -y_{max})$

Line segment intersection cases
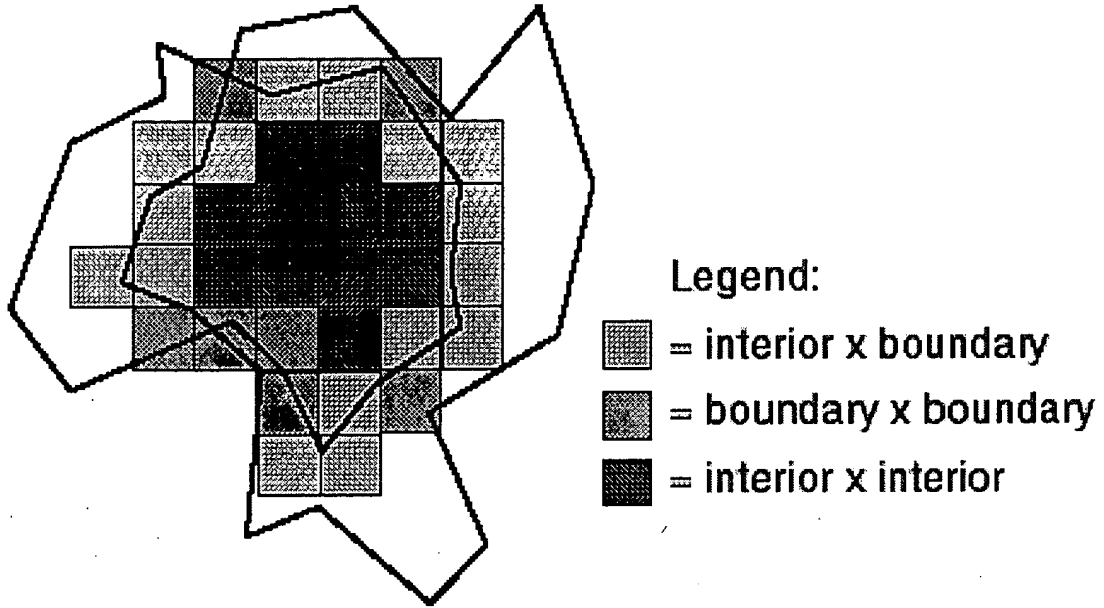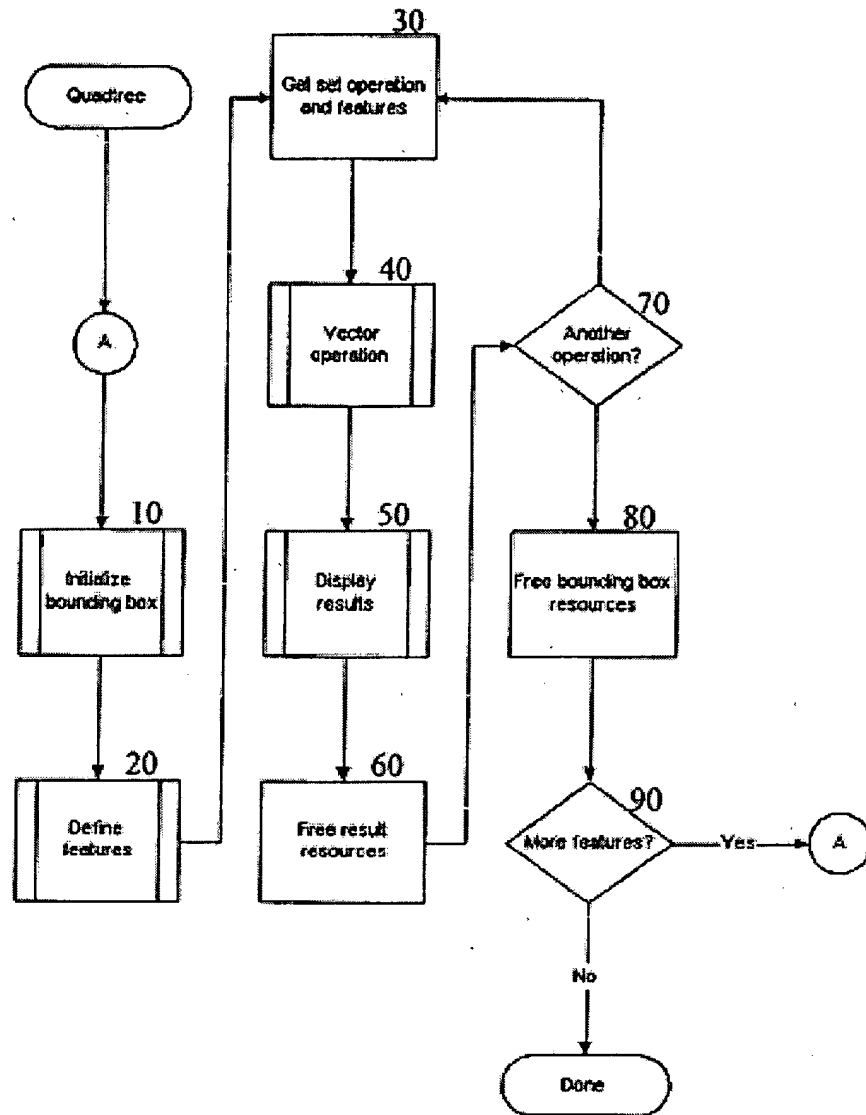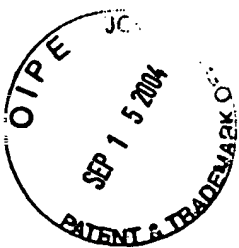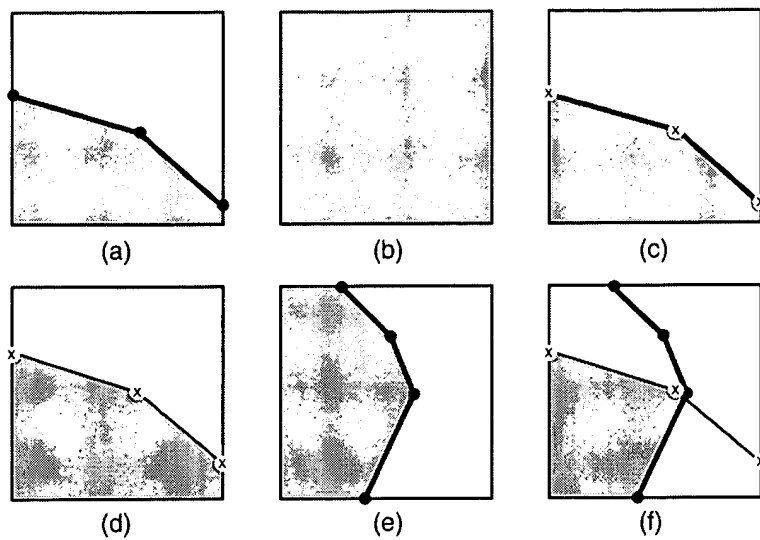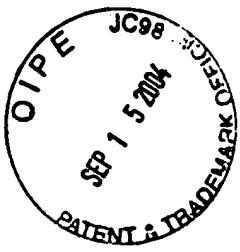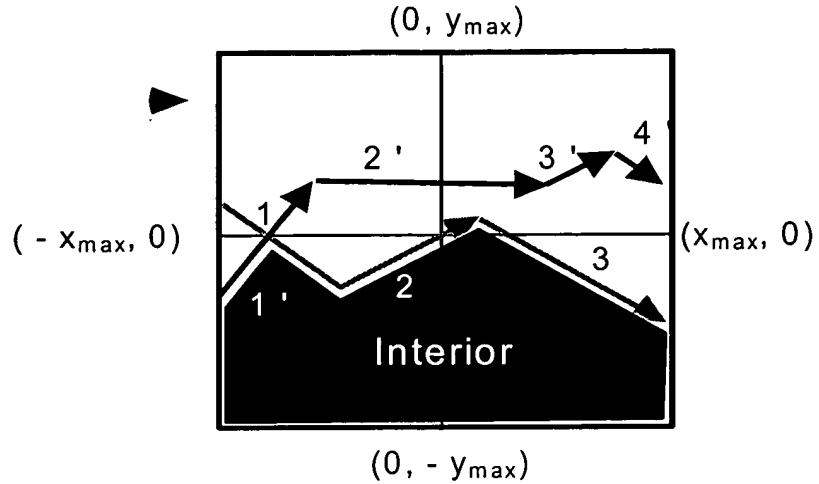
The two bounding boxes associated with the first two tuple-pairs from both features intersects as shown to the right

Case 1
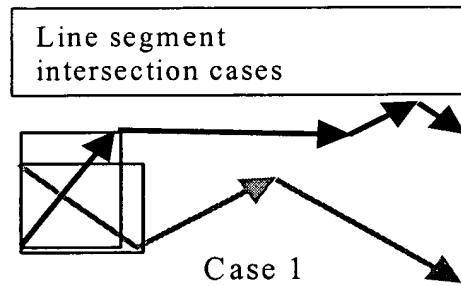
The bounding boxes associated with the second set of tuple-pairs do not intersect

Case 2
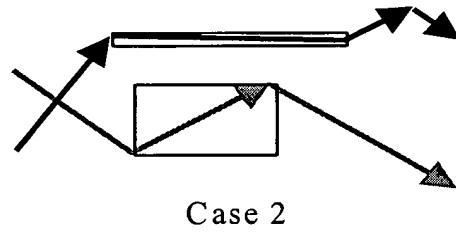
In the case to the right, two black line segments must be evaluated for intersection with a single red line segment.
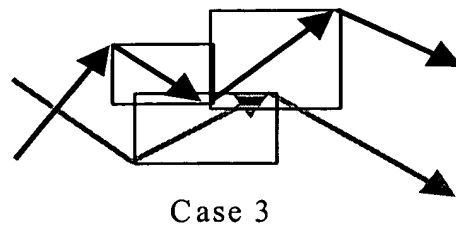
Case 3

Fig. 8.

Fig. 9.

No line intersection

Intersection at a point

Co-linear line segments

Crossing line segments

Fig. 10

1

1

Fig. 11

Gray 2

Black 2

Black 1

Gray 1

Fig. 12

4

1

2

3

Fig. 13

Possible cell entrance/exit point combinations

Disjoint

Enclosed

Partial overlap
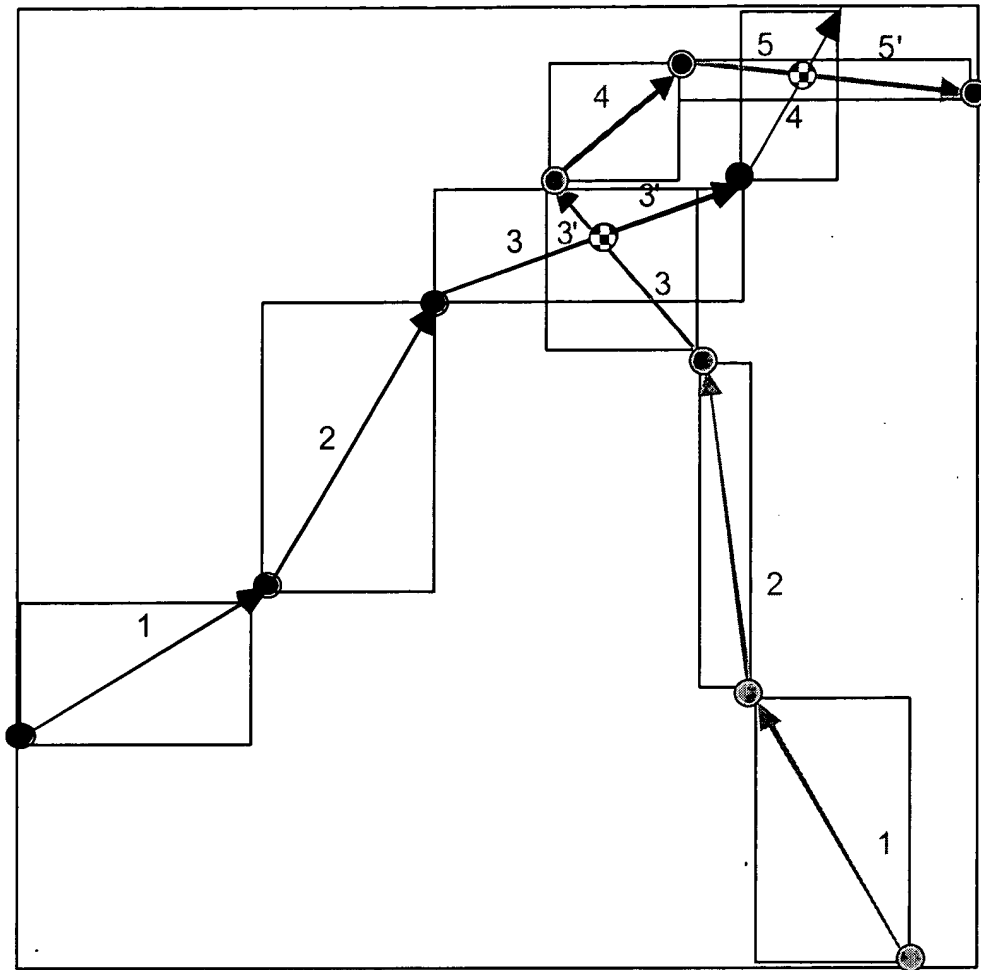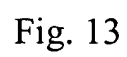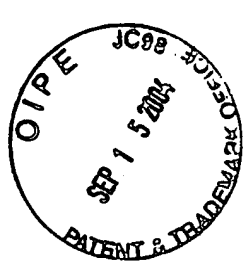
Fig. 14

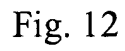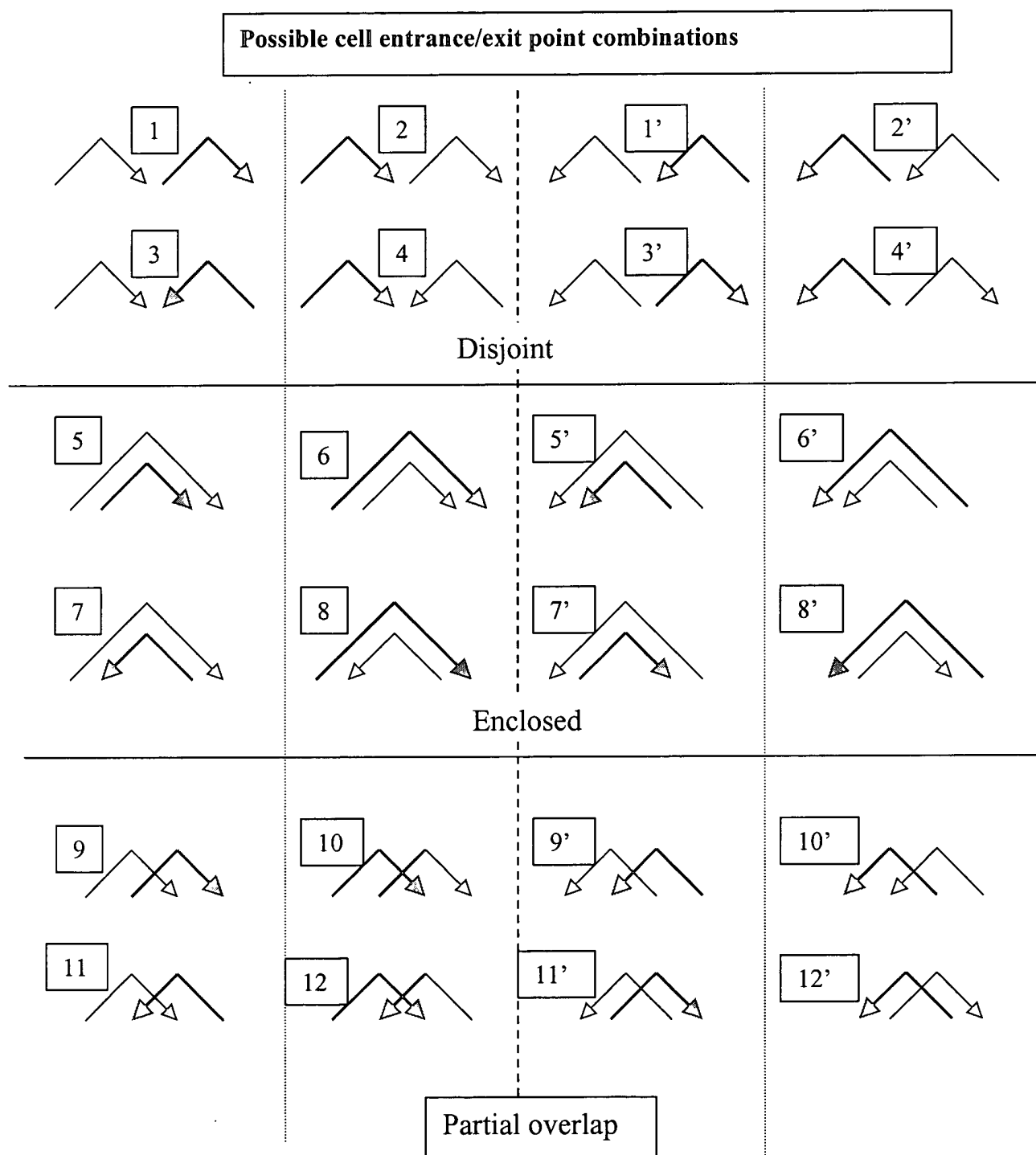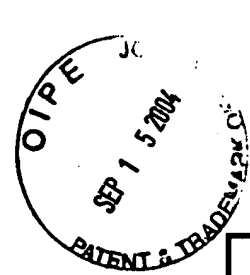| Entry/Exit Relationship	Class	Begin point for first cycle (inside is "to the right") Intersection	Begin point for first cycle (inside is "to the right") Union	Begin point for first cycle (inside "to the left") Intersection	Begin point for first cycle (inside "to the left") Union
$G_E G_X B_E B_X$	I	First intersection point of black or gray feature (Null if no intersect points)	Pseudo points (B$_E$ and G$_E$)	Pseudo points (B$_E$ and G$_E$)	First intersection point of black or gray feature (Full cell if no intersect points)
$G_E B_X B_E G_X$	II	Pseudo points (B$_E$ and G$_E$)	First intersection point of black or gray feature (Full cell if no intersect points)	First intersection point of black or gray feature (Null if no intersect points)	Pseudo points (B$_E$ and G$_E$)
$G_E B_E B_X G_X$	III	Pseudo point B$_E$	Pseudo point G$_E$	Pseudo point G$_E$	Pseudo point B$_E$
$G_E B_E G_X B_X$	VI	Pseudo point B$_E$	Pseudo point G$_E$	Pseudo point G$_E$	Pseudo point B$_E$
$G_E G_X B_X B_E$	IV	Pseudo point G$_E$	Pseudo point B$_E$	Pseudo point B$_E$	Pseudo point G$_E$
$G_E B_X G_X B_E$	V	Pseudo point G$_E$	Pseudo point B$_E$	Pseudo point B$_E$	Pseudo point G$_E$

Application of table:
Follow specified boundary entrance feature, accumulating intersection and/or union cycles until all polyline intersection point tuples in the cell have been exhausted.
Cycles alternate systematically along the specified entrance feature between contributions to intersection and union.
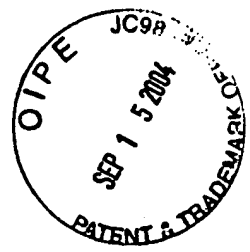Cycles are completed when they close on themselves. The implicit boundary-closing segments of a boundary-closing cycle are not actually represented in the generated product.

Comments:
As is apparent from the above formulation, intersection and union are effectively dual operations. The set operation generation procedure is similar regardless of the ordering convention of the polygon tuples (clockwise or counter-clockwise oriented), reflected in the symmetry observed within the above table.
Note that the classes are grouped into pairs. Classes I and II involve *inverse* operations; Classes III and VI employ identical generation operations, as do Classes IV and V.

Fig. 15

Fig. 16

Fig. 17
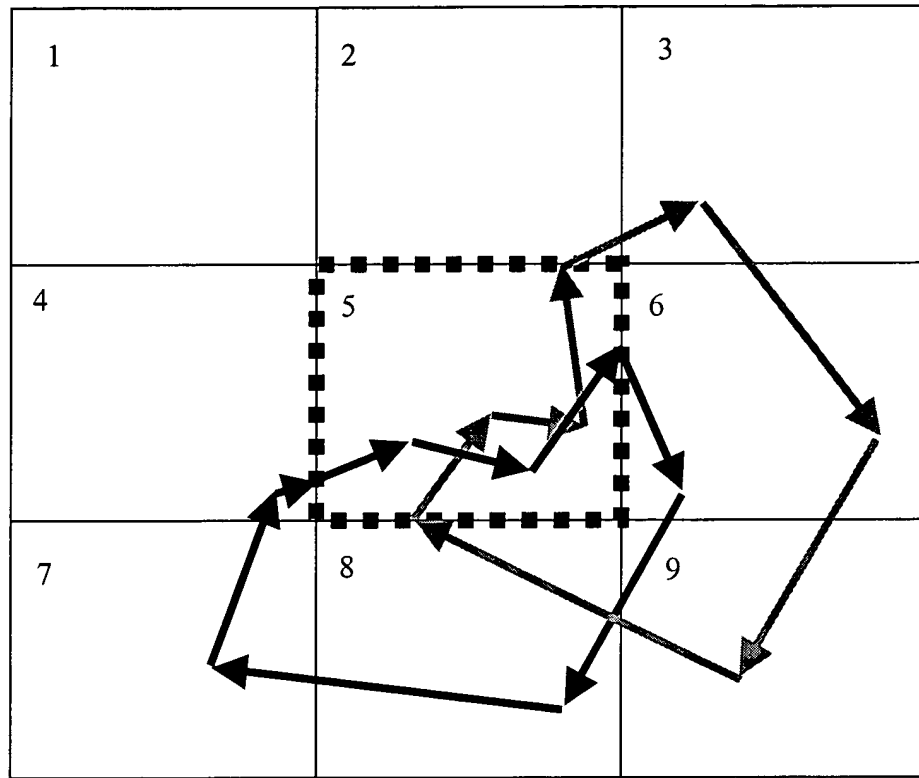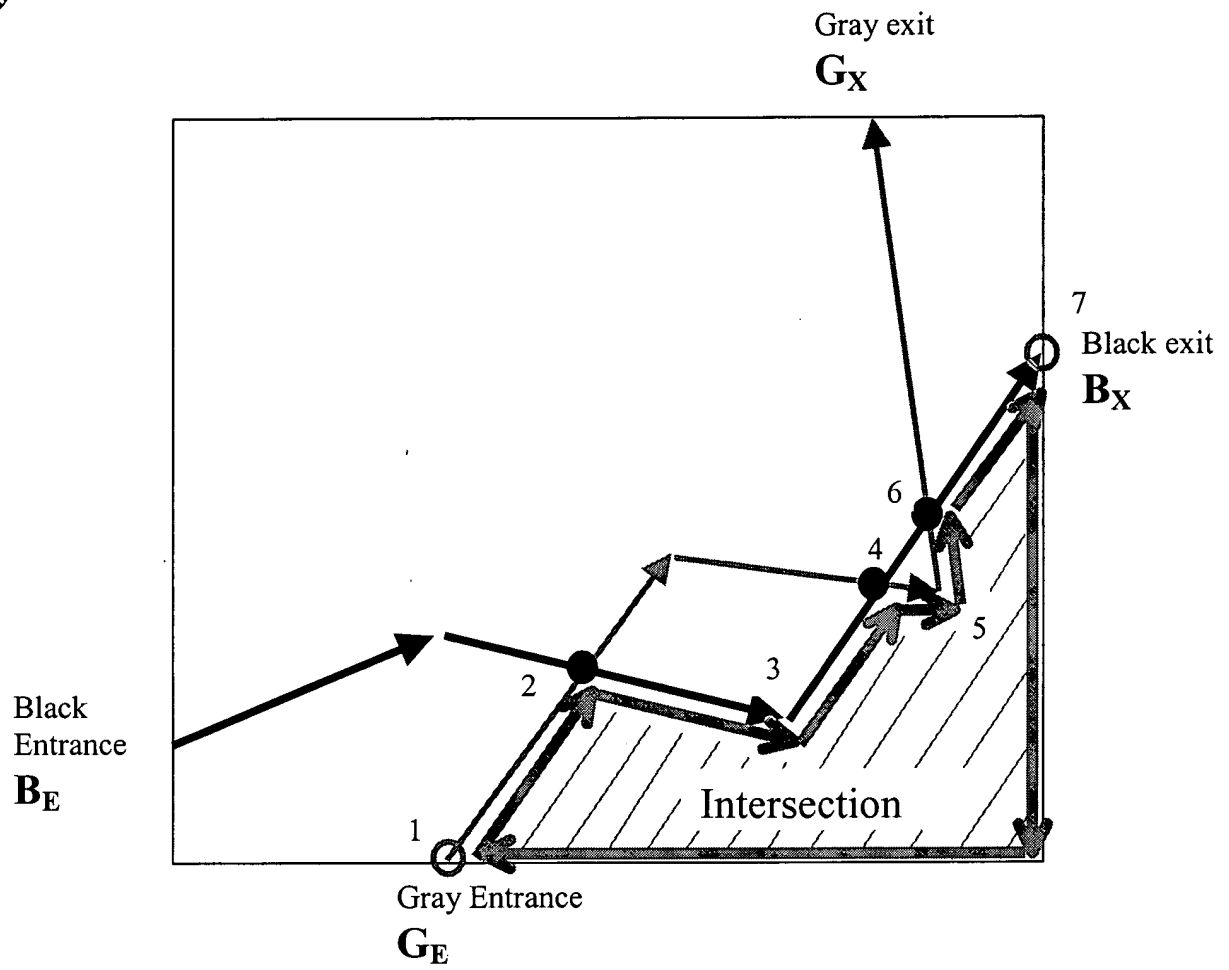
Fig. 18

Intersection products outlined in gray; union in black

B_X

G_X

Not union

G_E

B_E

8

10

9

3

4

5

6

7

2

1
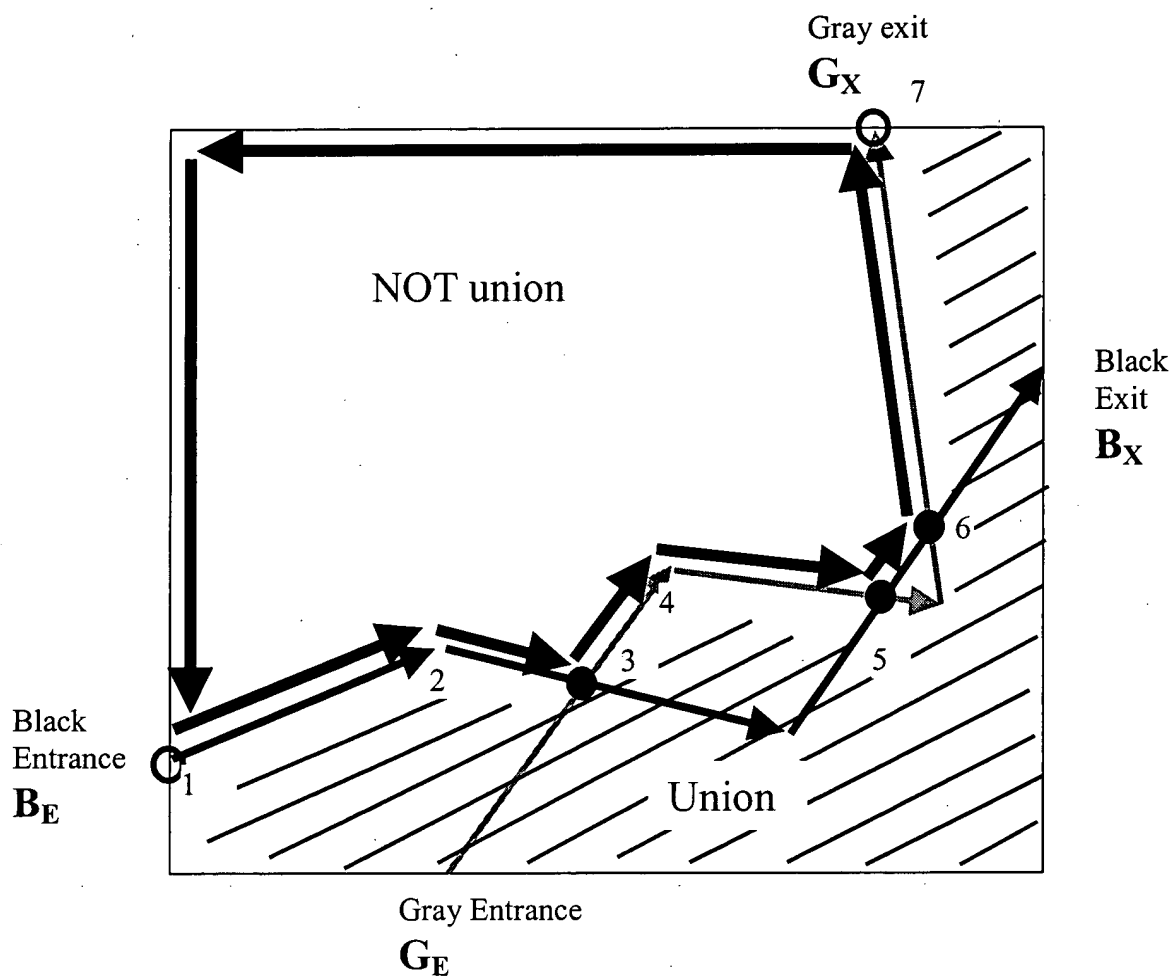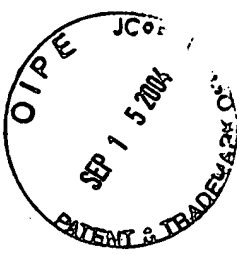
intersection

Class III:
$G_E B_E B_X G_X$
intersections=even

intersections=0
intersection cycles=1
union cycles=1

intersections=2
intersection cycles=1
union cycles=1

Fig. 19

B_X G_X

Not union

B_E

G_E

8

4

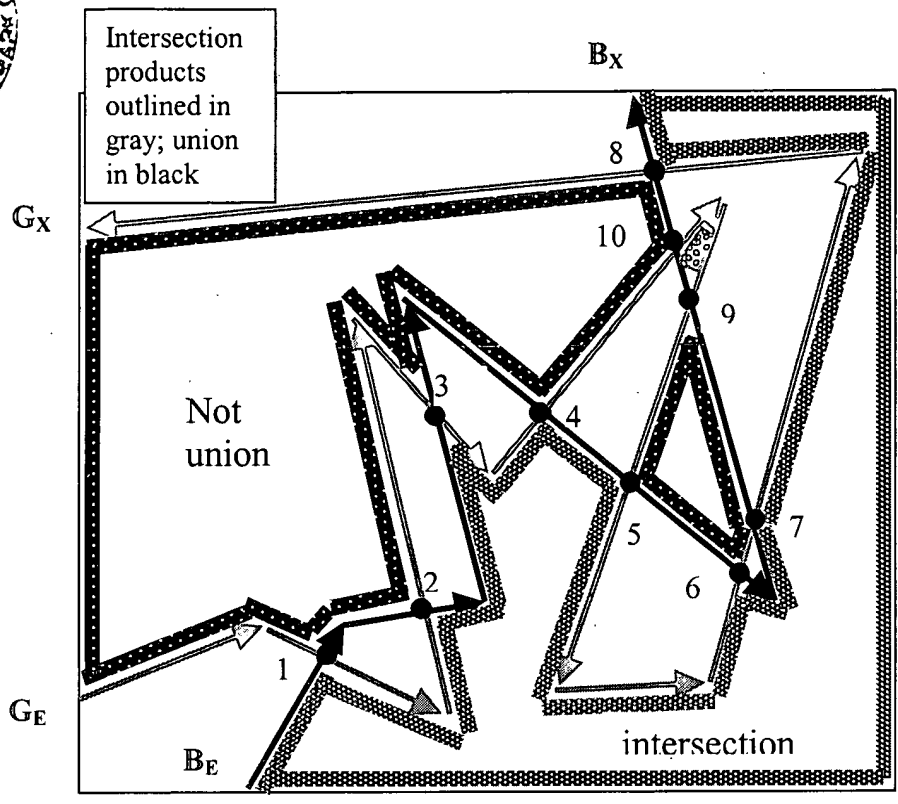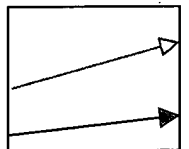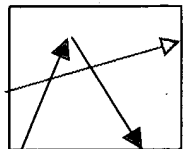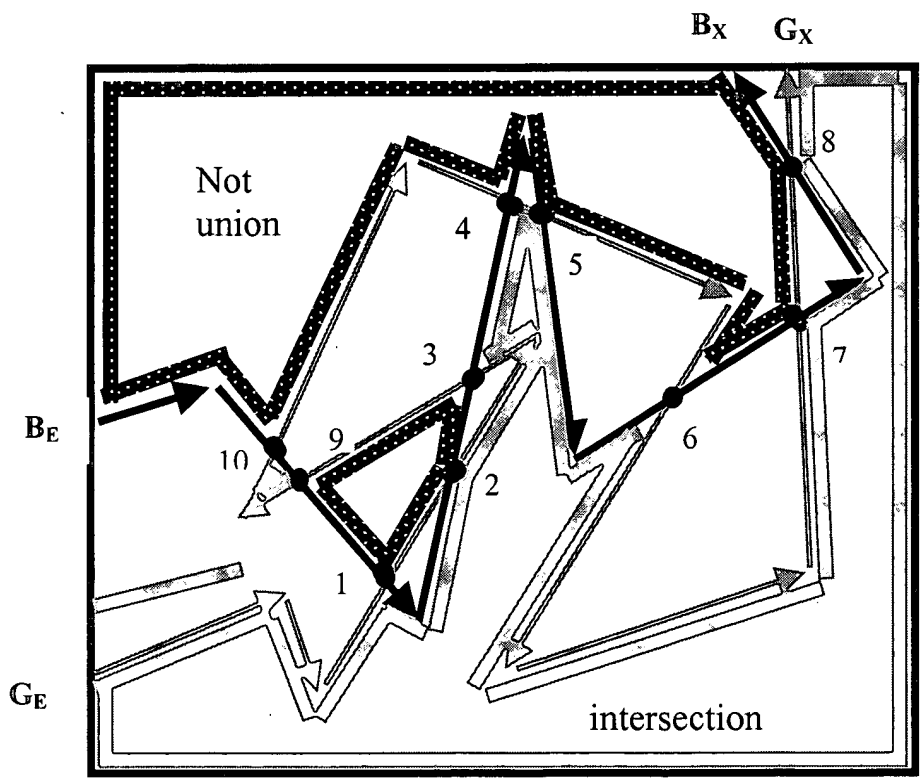5

3

9

10

2

1

6

7

intersection

Class IV:
$G_E G_X B_X B_E$
intersections=even

intersections=0
intersection cycles=1
union cycles=1

intersections=2
intersection cycles=1
union cycles= 1

Fig. 20

Fig. 21

In Fig. 21:
- B_E
- G_E
- Intersection products outlined in gray; union in black
- # intersections=3
- # inter. cycles=2
- # union cycles=2
- Class V:
  $G_E B_X G_X B_E$
  # intersections=odd
- # intersections=1
- # intersection cycles=1
- # union cycles=1
- intersection
- B_X
- G_E
- 1, 2, 3, 4, 5, 6, 7

Fig. 22

In Fig. 22:
- Not union
- # intersect=3
- # inter. cycles=2
- # union cycles=2
- Class VI:
  $G_E B_E G_X B_X$ □#
  intersections=odd□□
- # intersections=0
- # intersection cycle=1
- # union cycles=1 (or 2)
- B_E
- G_X
- G_E
- B_E
- intersection
- 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11

Intersection products outlined in gray; union in black

intersection

7

6

5

4

8
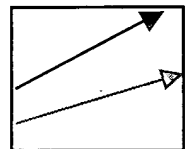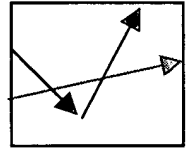
B_X

9

1

2

3

G_E

1'

G_X B_E

Fig. 23

Class I:
$G_E G_X B_E B_X$
intersect =even

intersections=0
intersection cycles=0
union cycles=1

intersections=2
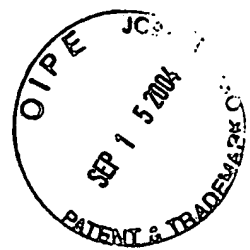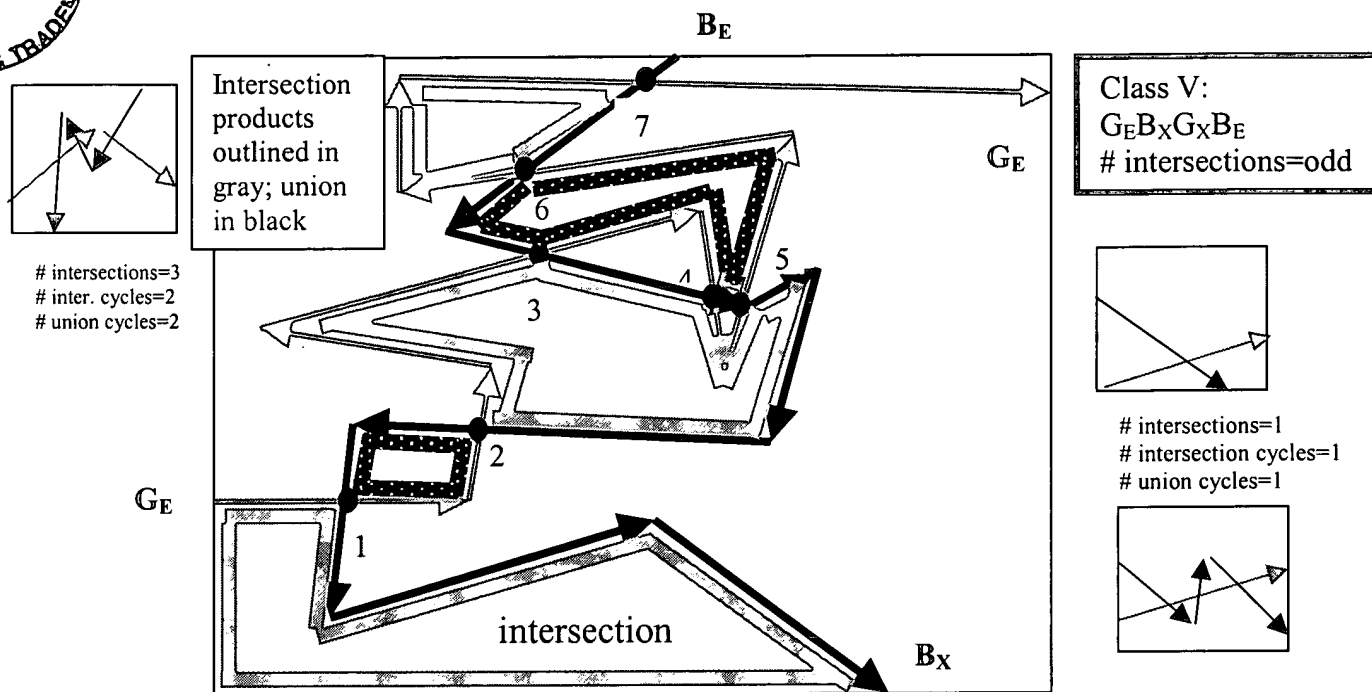# intersection cycles=1
union cycles=2

Intersection products outlined in gray; union in black

G_E intersect

B_E

5

4

2

6

3   1

7   Not union

8

9

12   10

11

G_E

B_X

Fig. 24.

Class II:
$G_E B_X B_E G_X$
intersections=even

intersections=0
intersection cycles=1
union cycles=all cell within

intersections=2
intersection cycles=2
union cycles=1

Class IV:
$G_E G_X B_X B_E$
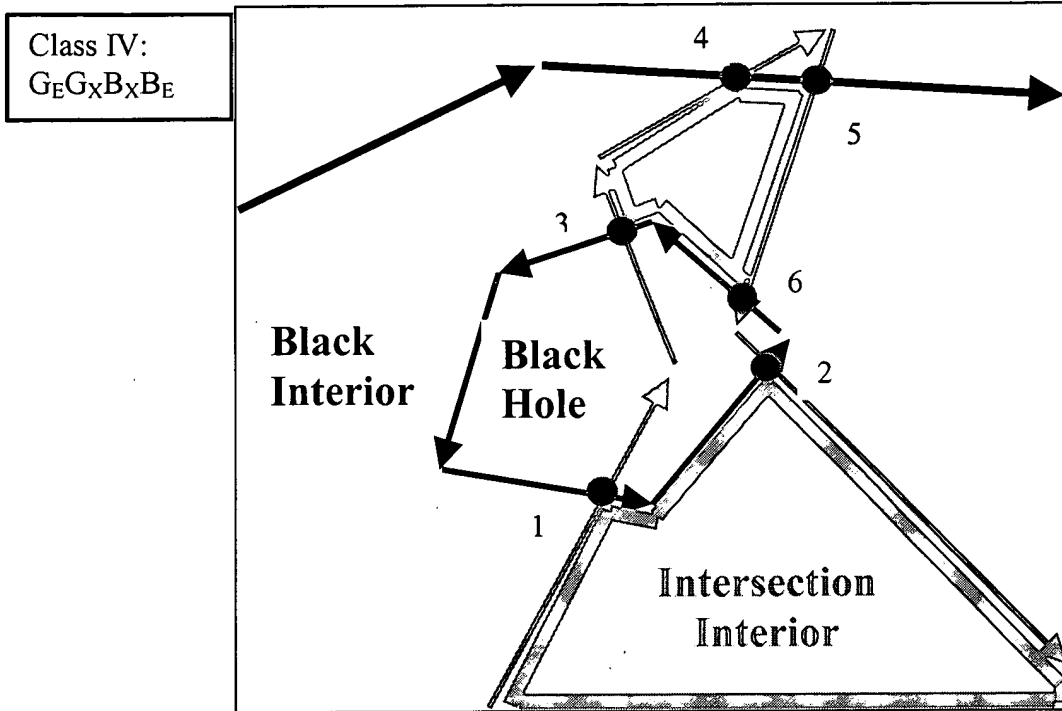
Fig. 25

Class I:
$G_E G_X B_E B_X$
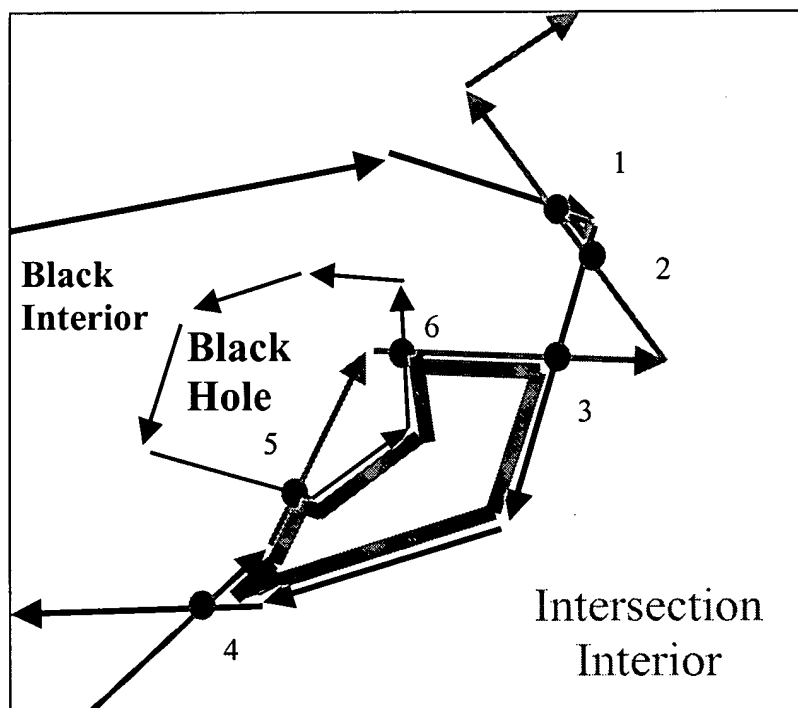
Fig. 26
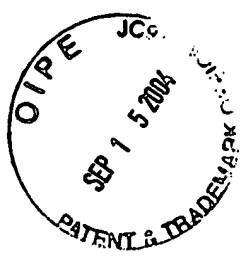
Class IV:
$G_EG_XB_XB_E$

Black
Interior

Black
Hole

Intersection
Interior

Fig.27

Both resultant
intersection
classes are
Class IV:
$G_FG_XB_XB_F$

Black
Interior

Black
Hole
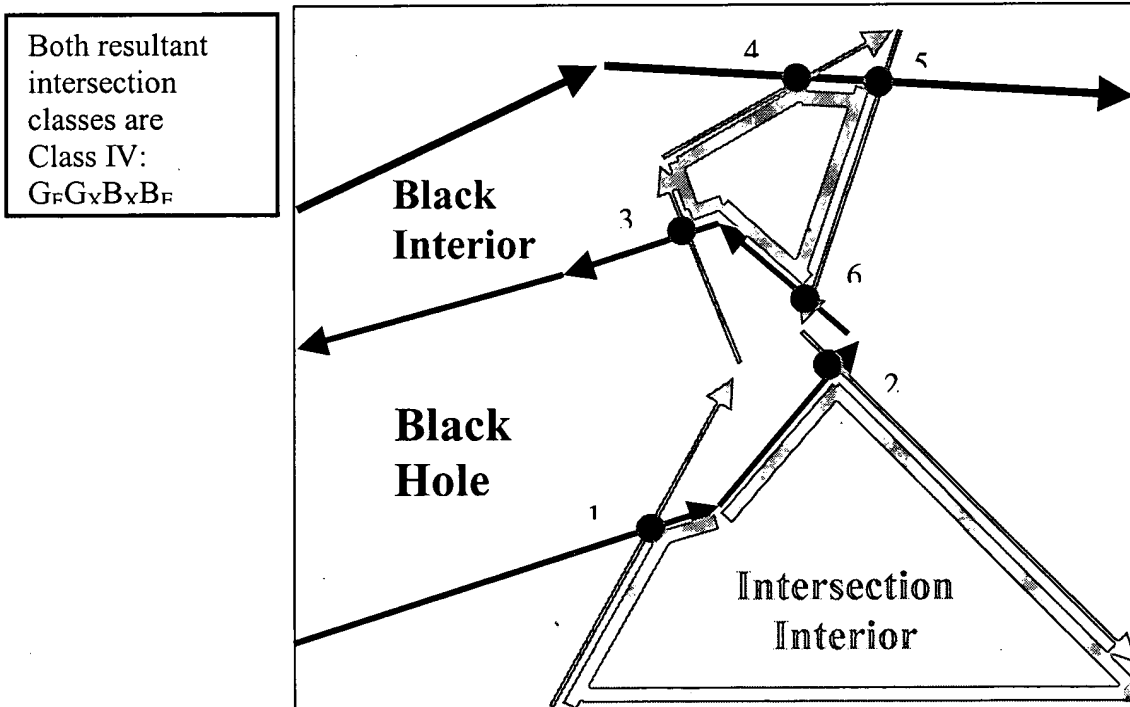
Intersection
Interior

Fig. 28

Entry/Exit Relationship	Class	Total # of Intersecti tuples within cell	Entry/exit relationship adjacent/ alternating	Special cases # intersect points	Total Intersect Cycles (to the right)	Total # of NOT union cycles (to the right)	Total Intersect cycles (to the left)	Total # of NOT union cycles (to the left)
$G_E G_X B_E B_X$	I	even	adjacent	0 2	0 1	2 2	1 1	full cell 2
$G_E B_X B_E G_X$	II	even	adjacent	0 2	1 2	full cell 1	0 2	2 1
$G_E B_E B_X G_X$	III	even	adjacent	0 2	1 1	1 1	1 1	1 1
$G_E B_E G_X B_X$	VI	odd	alternating	1	1	1	1	1
$G_E G_X B_X B_E$	IV	even	adjacent	0 2	1 1	1 1	1 1	1 1
$G_E B_X G_X B_E$	V	odd	alternating	1	1	1	1	1

Fig. 29

Fig. 30